Nearby resonances beyond the Breit-Wigner approximation

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Abstract

We consider a description of propagators for particle resonances which takes into account the quantum mechanical interference due to the width of two or more nearby states that have common decay channels, by incorporating the effects arising from the imaginary parts of the one-loop self-energies. Depending on the couplings to the common decay channels, the interference effect, not taken into account in the usual Breit-Wigner approximation, can significantly modify the cross section or make the more long-lived resonance narrower. We give few examples of New Physics models for which the effect is sizable, namely a generic two and multiple Higgs model and neutral vector resonances in Higgsless models. Based on these results we suggest the implementation of a proper treatment of nearby resonances into Monte Carlo generators.

1 Introduction

The Breit-Wigner (BW) approach to resonances in particle physics allows to take into account the finite width of a meta-stable particle. Various forms of this approach exist, which allow to describe different situations, from the narrow width approximation, up to broad and energy dependent widths.

In the following we consider a generalisation of the Breit-Wigner description [1] which makes use of a matrix propagator including non-diagonal width terms in order to describe physical examples in which these effects are relevant. Indeed for more than one meta-stable state coupled to the same particles, loop effects will generate mixings for the masses as well as mixed contributions for the widths (imaginary parts). In general a diagonalisation procedure for the masses (mass eigenstates) will leave non-diagonal terms for the widths.

Usually non-diagonal width terms are discarded. This is a good approximation in most cases and different unstable particles are described each by an independent Breit-Wigner profile. However when two or more resonances are close-by and have common decay channels (a precise definition of nearby resonances will be given in the following) such a description is not accurate anymore. Indeed, already from a quantum mechanical point of view, one cannot treat these states as independent. The usual Breit-Wigner approximation amounts to sum the modulus square of the various amplitudes neglecting the interference terms. When there are common decay channels and the widths of the unstable particles are of the same order of the mass splitting, the interference terms may be non-negligible. Generalisations of the Breit-Wigner approach were discussed in the literature in the past (see [1] for a general discussion of unstable-particle mixing, gauge invariance issues, unitarity and renormalisation for strongly-mixed systems), mainly focussing to the applications in the Higgs sector of the supersymmetric standard model and CP violation [2–6]. In the following we shall give a general formalism for scalar and vector resonances and discuss other relevant physical examples. In particular we will consider models of physics Beyond the Standard Model (BSM) in which new resonances play a crucial role, such as a generic two-Higgs model or the case of an arbitrary number of Higgs particles. Interesting is also the case of Higgsless models for which the lowest lying Kaluza-Klein (KK) excitations of the photon and the Z are nearly degenerate and have common decay channels into fermions and gauge bosons. We will show that in all these cases, the interference effects can play an important role. Based on these results we suggest that a proper treatment should be carefully implemented into Monte Carlo generators as physical results may be dramatically different from a naive use of the Breit-Wigner approximation.

2 Scalar fields

We first discuss the case of scalar fields which gives a simpler overview of the problem without the extra complications of the gauge and Lorentz structure present in other cases. The main observation needed to take the mixing effect into account, is to remember the link between the action and the propagator in quantum field theory and write the correction to

the propagator as a modification of the kinetic operator, which, in the general case, is a matrix.

Let's start with the action for a real scalar field given by (in position and momentum space):

$$\mathcal{L}_{\text{scalar}} = \int d^4x \, \frac{1}{2} \left[(\partial_\mu \phi)^2 - m_0^2 \phi^2 \right] = \int \frac{d^4p}{(2\pi)^4} \, \frac{1}{2} \phi(-p) K_{s,0}(p) \phi(p) \,, \tag{2.1}$$

where, for later convenience, we have defined a kinetic function

$$K_{s,0}(p) = p^2 - m_0^2 \,. \tag{2.2}$$

The propagator of the scalar field is defined as the inverse of the kinetic operator:

$$i\Delta_{s,0} = iK_{s,0}^{-1} = \frac{i}{p^2 - m_0^2} \,. \tag{2.3}$$

In the presence of interactions, the kinetic term will receive contributions by loops: if we call $i\Pi(p^2)$ the value of the 1-PI corrections to the propagator, the corrected kinetic term is

$$K_s = p^2 - m_0^2 + \Pi(p^2) \,. \tag{2.4}$$

Therefore the new propagator is

$$i\Delta_s = iK_s^{-1} = \frac{i}{p^2 - m_0^2 + \Pi(p^2)} = i\Delta_{s,0} \sum_{n=0}^{\infty} (-1)^n \left(\Pi \Delta_{s,0} \right)^n \,, \tag{2.5}$$

which corresponds to the resummation of the 1-PI insertions on the bare propagator. The pole of the resummed propagator defines, as usual, the renormalised mass. If the particle is unstable, $\Pi(p^2)$ is complex. The real part is used to renormalise the mass and the imaginary part defines the width of the particle. The propagator has a pole in $m^2 - im\Gamma$, with m the renormalised mass and Γ the width. For our purposes, we leave out the imaginary part

$$\Im \Pi(p^2) = \Sigma(p^2) \,. \tag{2.6}$$

In the narrow width approximation, $\Sigma(m^2) = m\Gamma$.

This formalism can be generalised to a system involving multi-fields, which do couple to the same intermediate particles: the loops will generate mixings in the masses, but also out-of-diagonal imaginary parts. In general the real and imaginary parts will not be diagonalisable at the same time: we are interested in the phenomenological consequences of this scenario, when the out-of-diagonal imaginary part is of the same order as the mass splitting. The kinetic function is now written in matrix form:

$$(K_s)_{lk} = (p^2 - m_l^2)\delta_{lk} + i\Sigma_{lk}(p^2) \,. \tag{2.7}$$

(We are considering the imaginary part only, the real one is used to renormalise the masses.) The propagator of the fields can be defined as the inverse of this matrix:

$$i(\Delta_s)_{lk} = i \left(K_s^{-1} \right)_{lk} \,. \tag{2.8}$$

To give an explicit example, let us focus on the two-particle case:

$$i\Delta_s = \frac{i}{D_s}\begin{pmatrix} p^2 - m_2^2 + i\Sigma_{22} & -i\Sigma_{12} \\ -i\Sigma_{21} & p^2 - m_1^2 + i\Sigma_{11} \end{pmatrix}\,, \tag{2.9}$$

where

$$D_s = (p^2 - m_1^2 + i\Sigma_{11})(p^2 - m_2^2 + i\Sigma_{22}) + \Sigma_{12}\Sigma_{21}\,. \tag{2.10}$$

For vanishing Σ_{12} and Σ_{21}, the propagator is diagonal and it reduces to two independent Breit-Wigner propagators with $m_i\Gamma_i = \Sigma_{ii}(m_i^2)$.

However, the narrow width approximation is not valid if the off-diagonal terms are sizable compared with the mass splitting. Defining $2M^2 = m_2^2 + m_1^2$ and $2\delta = m_2^2 - m_1^2$, the poles of the propagator (zeros of D_s), which define the physical masses and widths of the two resonances, are:

$$\tilde{m}_\pm^2 = M^2 - i\frac{\Sigma_{11}+\Sigma_{22}}{2} \pm \frac{i}{2}\sqrt{(\Sigma_{22}-\Sigma_{11}+2i\delta)^2 + 4\Sigma_{12}\Sigma_{21}}\,. \tag{2.11}$$

Note that the value of the masses is modified by the presence of the off-diagonal terms due to the imaginary part of the square root, at the same time the widths are affected. More importantly, the off-diagonal terms in the propagator will generate non-negligible interference, which can be in turn constructive or destructive. This effect will be illustrated with some numerical examples.

Finally, let us write some general formulae for $\Sigma(p^2)$. If we assume that the scalar particles i and j couple to a pair of particles α with couplings λ_α^i and λ_α^j respectively, the matrix Σ_{ij} can be written in general as

$$[\Sigma(p^2)]_{ij} = \sum_\alpha \lambda_\alpha^i \lambda_\alpha^j f_\alpha(p^2)\,. \tag{2.12}$$

We will consider couplings with fermions f, scalars s and vectors V:

$$\phi_i\left(\bar{f}(\lambda_{fL}^i P_L + \lambda_{fR}^i P_R)f' + \lambda_s^i s^\dagger s' + \lambda_V^i V_{1\mu}^\dagger V_2^\mu\right)\,, \tag{2.13}$$

where $P_{L,R} = (1 \mp \gamma_5)/2$ are the chirality projectors. Therefore, expanding for small masses of the particles in the loop (full results are given in appendix A), we get:

$$\left[\Sigma_f^{(s)}(p^2)\right]_{ij} = \frac{\lambda_{fL}^i\lambda_{fR}^j + \lambda_{fR}^i\lambda_{fL}^j}{16\pi}p^2 + \dots \tag{2.14}$$

$$\left[\Sigma_s^{(s)}(p^2)\right]_{ij} = \frac{\lambda_s^i\lambda_s^j}{16\pi} + \dots \tag{2.15}$$

$$\left[\Sigma_V^{(s)}(p^2)\right]_{ij} = \frac{\lambda_V^i\lambda_V^j}{64\pi}\frac{p^4}{m_{V_1}^2 m_{V_2}^2} + \dots \tag{2.16}$$

Note that the momentum dependence in the previous formula could give rise to violation of high-energy unitarity at energies above the resonance as well as distortion of the line-shape for broad resonances. These issues are discussed in detail in [1, 7]. Actually we do not face these problems here, since we are interested in the effects of the absorptive self-energies near the pole of nearby resonances whose widths are of the same order of magnitude of the mass splitting.

2.1 A numerical example: near-degenerate Higgses

As a numerical example, we will study two heavy Higgses where both the scalars develop a vacuum expectation value (VEV) and therefore couple to the W and Z gauge bosons. This situation is common in supersymmetric models where two Higgses are required by writing supersymmetric Yukawa interactions for up and down type fermions, and generic two Higgs models. The interference between near degenerate Higgses has been studied in [2–4] focusing in CP violation effects.

The couplings of the two CP-even Higgses to gauge bosons can be written as

$$\begin{aligned} \lambda_{WWH1} &= g\, m_W \cos\alpha\,, & \lambda_{WWH2} &= g\, m_W \sin\alpha\,, \\ \lambda_{ZZH1} &= \frac{g\, m_Z}{\cos\theta_W}\cos\alpha\,, & \lambda_{ZZH2} &= \frac{g\, m_Z}{\cos\theta_W}\sin\alpha\,; \end{aligned} \tag{2.17}$$

where α is a mixing angle taking into account the mixing between the two mass eigenstates and the difference between the two VEVs.

Here we are interested in a generic production cross section of the two nearby Higgses on the resonances, with decay of the Higgses into gauge bosons (either WW or ZZ). The amplitude of this process is proportional to the resonant propagator weighted by the couplings given in eq.(2.17). In the case we are considering, the common decay channels can give off-diagonal terms in eq.(2.9) which are sizable compared with the mass splitting. Therefore we need to include their effects and a generic cross section will be proportional to (here we assume that the coupling to the initial particles are the same):

$$\left|(\Delta_s^{11} + \Delta_s^{21})\cos\alpha + (\Delta_s^{22} + \Delta_s^{12})\sin\alpha\right|^2 . \tag{2.18}$$

In Fig. 1, we plot this quantity in arbitrary units and compare it with the Breit-Wigner approximation: we fix $m_{H1} = 400$ GeV, and vary the splitting from 50 to 5 GeV. For simplicity, in the following we will assume $\alpha = \pi/4$, so that the two scalars have the same couplings (but this assumption is not crucial for our conclusions). The exact treatment of the resonances unveils a destructive interference (which is neglected in the naive Breit-Wigner case) that can drastically reduce the cross section. Also, the interference between the two resonances splits the mass poles [3, 4].

This effect can be even more important for scenarios with a large number of scalars as predicted in some string models. Our analysis can be easily extended to an arbitrary number of Higgses. Let's take for example the couplings to the gauge bosons to be given by $g_{\rm SM}/\sqrt{N}$, where g_{SM} is the SM coupling of the gauge bosons and N is the number of Higgses.

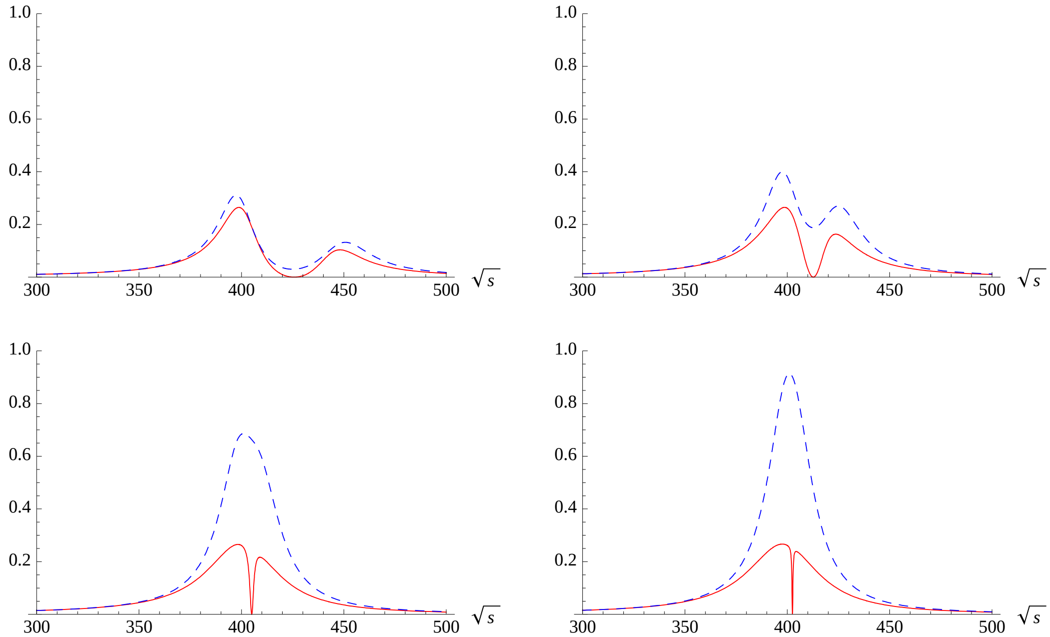

Figure 1: Plots of the production cross section (in arbitrary units) of two nearby Higgses decaying into gauge boson pairs for the naive Breit-Wigner (blue-dashed) and exact mixing (red-solid). The mass of the first resonance is fixed to 400 GeV, the splitting respectively 50, 25, 10 and 5 GeV and $\alpha = \pi/4$.

In Fig. 2 we plot the cross section for seven nearby Higgses, with the first one at 400 GeV and the others at a distance of 5 and 10 GeV, the width of each being 6.2 GeV. From the plot it is clear that the destructive interference reduces the giant resonance (which is not distinguishable from a single Higgs, once the experimental smearing is taken into account) to a bunch of *gnometti* (dwarfs), which will be very hard to detect. The cross section is in fact reduced by a significant factor with respect to the naive expectation, and the smearing will wash out the peak structure. This situation is different from the continuum Higgs spectrum of [8], where the Higgs peaks are smeared by the experimental uncertainties only. Therefore, in this case, together with the appearance of a "continuum", the cross section is suppressed by the interference. It is intriguing to compare this analysis with Un-Higgs models [9, 10], where the Higgs in indeed a continuum: such behaviour may arise from the superposition of Kaluza-Klein resonances in extra dimensional realisations or deconstructed models [10].

3 Vector fields

For a vector field the technique is similar to the scalar case, however a major issue is defining the propagator of a metastable particle in a gauge invariant way (see for example [11]). Nevertheless, at the pole, only the values of the pole mass and width (which are gauge-invariant) are relevant, and extra terms that must be added to preserve gauge invariance are

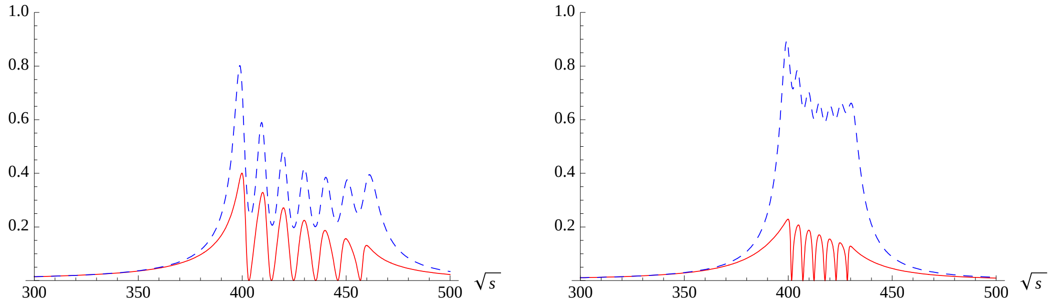

Figure 2: Plots of the production cross section (in arbitrary units) for seven nearby Higgses equally coupled to SM gauge bosons: the naive Breit-Wigner (blue-dashed) bump reduces to a row of seven dwarfs when the exact mixing (red-solid) is taken into account. The mass of the first resonance is fixed to 400 GeV, the splitting between the six Higgses respectively 10 and 5 GeV.

numerically negligible. The kinetic function for a vector V, in generic ξ-gauge, is

$$\begin{aligned} K_{\mu\nu,0} &= g_{\mu\nu}(p^2 - M_0^2) - p_\mu p_\nu \left(\frac{1}{\xi} - 1\right) \\ &= \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)(p^2 - M_0^2) + \frac{p_\mu p_\nu}{p^2}\left(\frac{p^2}{\xi} - M_0^2\right) . \end{aligned} \tag{3.1}$$

The propagator, is then defined as the inverse of the kinetic term:

$$\begin{aligned} i\Delta_{\mu\nu,0} &= -i\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)(p^2 - M_0^2)^{-1} - i\frac{p_\mu p_\nu}{p^2}\left(\frac{p^2}{\xi} - M_0^2\right)^{-1} \\ &= \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{-i}{p^2 - M_0^2} + \frac{p_\mu p_\nu}{p^2}\frac{-i\xi}{p^2 - \xi M_0^2} . \end{aligned} \tag{3.2}$$

The first term of the propagator has a gauge-independent pole at the V mass, while the other part has a gauge-dependent pole, which will cancel the pole given by the Goldstone boson (whose mass is indeed ξM_0^2). Therefore, at the pole we can neglect the contribution of the second term, and the propagator simplifies to

$$i\Delta_{\mu\nu,0} \simeq \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{-i}{p^2 - M_0^2} ; \tag{3.3}$$

which is equal to a Lorentz tensor times a scalar propagator.

Factorising out the Lorentz structure, loop corrections can be parameterised as

$$\Pi_{\mu\nu} = \Pi_T\, g_{\mu\nu} + \Pi_L\, p_\mu p_\nu , \tag{3.4}$$

so that

$$K_{\mu\nu,0} + \Pi_{\mu\nu} = \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)(p^2 - M_0^2 + \Pi_T) + \frac{p_\mu p_\nu}{p^2}\left(\frac{p^2}{\xi} - M_0^2 + \Pi_T + p^2\Pi_L\right) . \tag{3.5}$$

The corrected propagator is therefore:

$$i\Delta_{\mu\nu} = \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{-i}{p^2 - M_0^2 + \Pi_T} + \frac{p_\mu p_\nu}{p^2} \frac{-i\xi}{p^2 - \xi(M_0^2 - \Pi_T - p^2 \Pi_L)} \,. \tag{3.6}$$

The first term defines the pole mass and width (gauge independent), while the second term contains a gauge-dependent pole, and it is negligible at the physical pole. Here we will be interested only in the imaginary part $\Im \Pi_T = \Sigma$, which defines the decay width of the vectors. Neglecting the second part of the propagator, we find:

$$i\Delta_{\mu\nu} = \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) (-i) \left(p^2 - M_V^2 + i\Sigma \right)^{-1} = \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) (-i) \Delta_s(M_V) \,. \tag{3.7}$$

For a generic number of vectors, the same discussion as in the scalar case applies, and Δ_s has a matrix form, like in (2.8).

We conclude with some general formulae for Σ which can be expressed as in eq. (2.12). We will consider couplings with fermions f, scalars s and vectors V:

$$V_i^\mu \left(\bar{f} \gamma_\mu (\lambda_L^i P_L + \lambda_R^i P_R) f' + \lambda_s^i (q_1 - q_2)^\mu s_1^\dagger s_2 + \lambda_V^i V_1^\nu V_2^\rho G_{\mu\nu\rho} \right) \,, \tag{3.8}$$

where $G_{\mu\nu\rho} = g_{\mu\nu}(p + q_1)_\rho + g_{\nu\rho}(q_2 - q_1)_\mu - g_{\rho\mu}(q_2 + p)_\nu$, with $p = q_1 + q_2$. Expanding for small masses of the particles in the loop (full results are given in Appendix A) we obtain:

$$\left[\Sigma_f^{(V)}(p^2) \right]_{ij} = \frac{\lambda_L^i \lambda_R^j + \lambda_R^i \lambda_L^j}{24\pi} p^2 + \dots \tag{3.9}$$

$$\left[\Sigma_s^{(V)}(p^2) \right]_{ij} = \frac{\lambda_s^i \lambda_s^j}{48\pi} p^2 + \dots \tag{3.10}$$

$$\left[\Sigma_V^{(V)}(p^2) \right]_{ij} = \frac{\lambda_V^i \lambda_V^j}{192\pi} \frac{p^6}{m_{V_1}^2 m_{V_2}^2} + \dots \tag{3.11}$$

In the following we apply this formalism to a simple case.

3.1 Numerical example: Z' and A' in Higgsless models

In Higgsless models [12,13], the first two neutral resonances are nearly degenerate, and they correspond to the first KK excitation of the Z and of the photon. Here we will explicitly refer to the warped extra dimensional model in [13]: the masses can be approximated by

$$m_{Z'}^2 \simeq m_{KK}^2 + 4 m_Z^2 \,, \qquad m_{A'}^2 \simeq m_{KK}^2 \,, \tag{3.12}$$

so that the mass difference is very small:

$$m_{Z'} - m_{A'} \simeq 2 \frac{m_Z^2}{m_{KK}} \sim 16\,\mathrm{GeV} \cdot \left(\frac{1\mathrm{TeV}}{m_{KK}} \right)^2 \,. \tag{3.13}$$

In terms of the parameters of the warped geometry (R is the curvature, R' the position of the Infra-Red brane in covariant coordinates):

$$m_{KK} \sim \frac{2.4}{R'}\,, \qquad m_W = \frac{1}{R' \log \frac{R'}{R}}\,; \tag{3.14}$$

therefore, given the value of the curvature R, the KK mass (R') is determined by the W mass.

The coupling to the light gauge bosons can be estimated by use of the Higgsless sum rules, that ensure the cancellation of the terms growing like the energy square in the elastic scattering amplitude of the longitudinal W and Z, thus delaying the violation of perturbative unitarity at higher scales than in the Standard Model without Higgs. One of these sum rules is [14, 15]:

$$g_{WWWW} = \frac{3}{4}\left(g_{WWZ}^2 \frac{m_Z^2}{m_W^2} + \sum_k g_{WWk}^2 \frac{m_k^2}{m_W^2} \right) + \frac{g^2}{4}(1-\zeta)\,, \tag{3.15}$$

where $g_{WWWW} = g^2$, $g_{WWZ} = g \cos\theta_W$ and we have included a partial contribution for the Higgs ($\zeta = 1$ corresponds to the Higgsless limit, $\zeta \to 0$ to the SM Higgs). Neglecting the contribution of the heavier states, from eq.(3.15) we can estimate the coupling of the first tier:

$$g_{WW1} \simeq \sqrt{\frac{\zeta}{3}} g \frac{m_W}{m_{KK}}\,. \tag{3.16}$$

This is actually the coupling of the first KK mode of the neutral component in the SU(2) multiplet: therefore, in order to obtain the couplings of the mass eigenstates, one needs to impose a rotation by an angle θ_1, which describes the mixing between the SU(2) and the U(1) vectorial component:

$$g_{WWZ'} \simeq g_{WW1} \cos\theta_1\,, \qquad g_{WWA'} \simeq g_{WW1} \sin\theta_1\,. \tag{3.17}$$

Numerically, it turns out that the Z' is 45% in the SU(2) and 55% in U(1), therefore $\cos\theta_1 \sim \sqrt{45\%} \sim 0.67$ and $\sin\theta_1 \sim \sqrt{55\%} \sim 0.74$.

The amplitude for the decay $Z', A' \to WW$ is given by ($\alpha_W = g^2/4\pi$):

$$\Sigma(p^2) \simeq \begin{pmatrix} \cos^2\theta_1 & \cos\theta_1\,\sin\theta_1 \\ \cos\theta_1\,\sin\theta_1 & \sin^2\theta_1 \end{pmatrix} \frac{\zeta\alpha_W}{144} \frac{p^6}{m_W^2 m_{KK}^2}\,; \tag{3.18}$$

and the width can be estimated by:

$$\Gamma = \frac{\Sigma(m_{KK}^2)}{m_{KK}} \sim \begin{pmatrix} 17 & 19 \\ 19 & 21 \end{pmatrix} \mathrm{GeV} \cdot \left(\frac{m_{KK}}{1\mathrm{TeV}}\right)^3 \zeta\,. \tag{3.19}$$

The off-diagonal entries are large, and they are of the same order of the mass splitting between the two neutral gauge bosons.

R	$m_{W'}$	$m_{A'}$	$m_{Z'}$	$g_{WWA'}$	$g_{WWZ'}$	$g_{f\bar{f}A'}$	$g_{f\bar{f}Z'}$
10^{-15}	1032	1028.5	1043.79	0.022	0.019	-0.021	0.065
10^{-10}	805.3	801.0	820.9	0.028	0.024	-0.027	0.085
10^{-7}	634.6	629.0	655.0	0.037	0.029	-0.035	0.11

Table 1: Three Higgsless points for the model proposed in [12], masses in GeV.

The determination of the couplings to fermions, which are relevant for the Drell-Yan production, is more involved. In fact, the typical scenario is that the right-handed components of the light quarks and leptons are localised on the Ultra-Violet brane, while the left-handed components are spread in the bulk [13]. For the left-handed couplings, we have some freedom: however, electroweak precision measurements prefer small couplings therefore we will neglect them. The couplings of the right-handed components depend uniquely on the suppression of the wave functions on the UV brane. Therefore, we can approximate:

$$g_{f\bar{f}1} \simeq Q_f g\, \tan\theta_W \, \frac{1}{\sqrt{\log\frac{R'}{R}}} \sim Q_f g\, \tan\theta_W \; 2.4 \frac{m_W}{m_{KK}} \sim Q_f \cdot 0.066 \cdot \left(\frac{1\text{TeV}}{m_{KK}}\right) . \qquad (3.20)$$

Numerically we find that:

$$g_{f\bar{f}Z'} = g_{f\bar{f}1}\,, \qquad g_{f\bar{f}A'} = g_{f\bar{f}1}\eta\,; \qquad (3.21)$$

were the ratio $\eta \sim -0.32$ is to a good approximation independent on the value of the KK mass. In Table 1, three numerical examples are given, corresponding to different values of the curvature: the masses and couplings are calculated exactly, following [12], and confirm our estimates.

We are interested in three processes: Drell-Yan production and decay into gauge bosons W^+W^- (DY), Drell-Yan production and decay into a pair of leptons (Leptonic) and vector boson fusion production followed by decay into gauge bosons (VBF). As in the scalar case, the amplitudes of these processes on resonance are proportional to the propagators weighted by the couplings with the incoming and outcoming particles. The cross section $\sigma(q\bar{q} \to A', Z' \to W^+W^-)$ is proportional to:

$$\sigma_{\text{DY}} \simeq \left| \cos\theta_1\, \Delta_s^{Z'Z'} + \sin\theta_1 \Delta_s^{Z'A'} - |\eta| (\sin\theta_1 \Delta_s^{A'A'} + \cos\theta_1\, \Delta_s^{A'Z'}) \right|^2 , \qquad (3.22)$$

which must be compared to the naive sum of two Breit-Wigner distributions:

$$\sigma_{\text{DY}}^{\text{naive}} \simeq \left| \cos\theta_1\, \Delta_s^{Z'Z'} - |\eta| \sin\theta_1\, \Delta_s^{A'A'} \right|^2 . \qquad (3.23)$$

On the other hand, the leptonic cross section $\sigma(q\bar{q} \to A', Z' \to l^+l^-)$ is proportional to:

$$\sigma_{\text{Leptonic}} \simeq \left| \eta^2\, \Delta_s^{A'A'} + \Delta_s^{Z'Z'} - |\eta|\, (\Delta_s^{Z'A'} + \Delta_s^{A'Z'}) \right|^2 ; \qquad (3.24)$$

$$\sigma_{\text{Leptonic}}^{\text{naive}} \simeq \left| \eta^2\, \Delta_s^{A'A'} + \Delta_s^{Z'Z'} \right|^2 . \qquad (3.25)$$

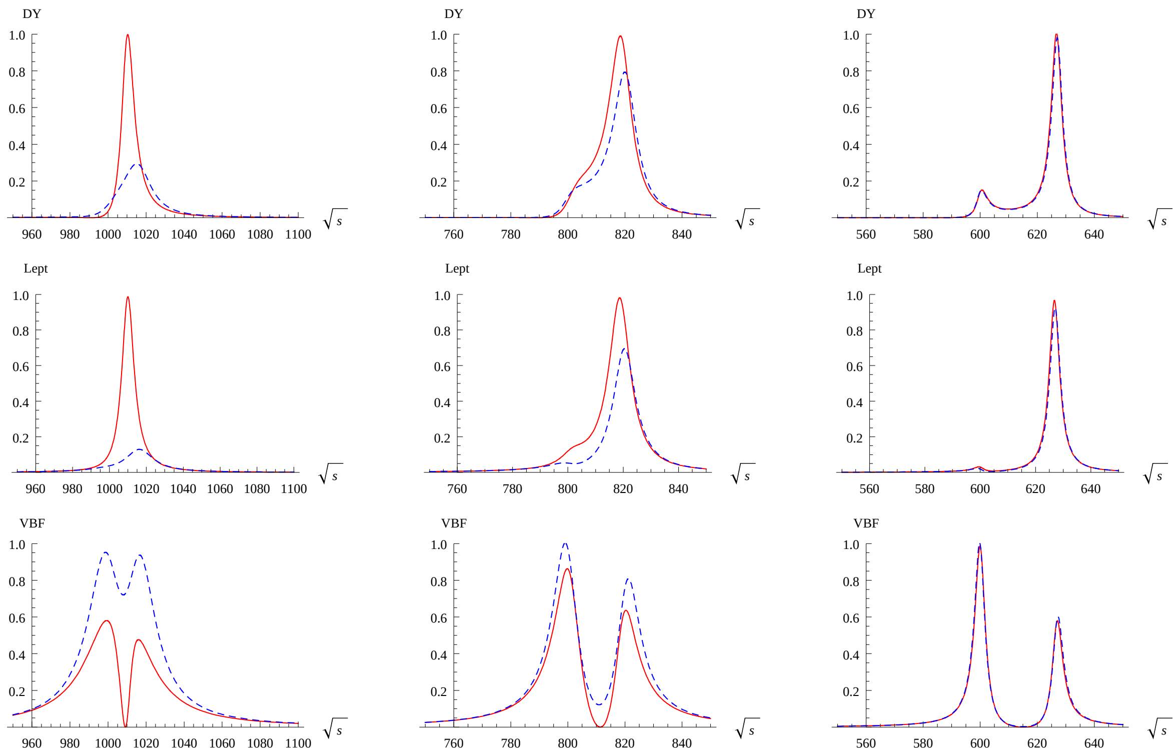

Figure 3: Plots of production cross section (in arbitrary units) of the two low-lying neutral resonances of the Higgsless model for the naive Breit-Wigner (blue-dashed) and exact mixing (red-solid). The rows correspond (from top to bottom) to DY, Leptonic and VBF; the columns (from left to right) correspond to $m_{KK} = 1000$ GeV, 800 GeV and 600 GeV.

Finally, for the VBF channel we have:

$$\sigma_{VBF} \simeq \left|\sin^2\theta_1\, \Delta_s^{A'A'} + \cos^2\theta_1\, \Delta_s^{Z'Z'} + \cos\theta_1\, \sin\theta_1\, (\Delta_s^{Z'A'} + \Delta_s^{A'Z'})\right|^2 ; \qquad (3.26)$$

$$\sigma_{VBF}^{\rm naive} \simeq \left|\sin^2\theta_1\, \Delta_s^{A'A'} + \cos^2\theta_1\, \Delta_s^{Z'Z'}\right|^2 . \qquad (3.27)$$

DY and Leptonic cross sections are easily calculable; the VBF channel requires a numerical study or a more involved approximate analytical expression.

In Figure 3 we plot, for illustrative purposes, the squared matrix element of the three resonant production channels for A' and Z' as function of $\sqrt{s}$ for three different cases: $m_{KK} = 1000$ GeV, 800 GeV and 600 GeV. For large masses, the effect of the interference is very important and it can affect the value of the cross section significantly. To make this more clear, we give hereafter the ratio of the area under the peaks in the figure obtained by the exact formula and the BW case. This roughly corresponds to the ratio of the integrated cross sections.

$M_{KK} =$	1000 GeV	800 GeV	600 GeV
DY:	1.6	1.15	1.02
Lept:	3.15	1.4	1.05
VBF:	0.6	0.8	0.97

In the VBF channel there can be a reduction up to 50%, while in the other two channels the interference is constructive and the total cross section can be enhanced by a factor of 2–3. The interference is therefore extremely important, especially in the TeV region. Since this represents the upper bound for Higgsless models, the interference effects are crucial to determine if the whole Higgsless parameter space can be probed at the LHC. As a consequence the implementation of the exact matrix propagator in the Monte Carlo generators seems mandatory for a correct analysis.

4 Conclusions

We have shown that for two or more unstable particles, when there are common decay channels and the masses are nearby, the interference terms may be non-negligible. This fact is well known in the literature and well studied since more than a decade [1], here we provided further examples in which this formalism is important. This kind of scenario is not uncommon in models of New Physics beyond the Standard Model, especially in models of dynamical electroweak symmetry breaking or in extended Higgs sectors. We reviewed the formalism for scalar and vector fields based on a matricial form of the propagator for multi-particles taking care of this case. This formalism can be easily extended to fermionic resonances as discussed in [1] and applied to heavy neutrino mixing in [16]. We gave few examples in which the effect of the non-diagonal width is important in physical results. In models with multi-Higgses and in Higgsless models with near degenerate neutral vector resonances, we showed that interference induced by the off-diagonal propagators are very important and they can either suppress or enhance the total cross sections on resonance depending on the relative sign of the couplings to the initial and final states. Similar issues were discussed in the CP violating MSSM for the Higgs sector in [17] and [18]. We expect similar effects in all the New Physics schemes involving resonances for which the mass splitting is of the same order of the off-diagonal matrix elements in the propagator due to common decay channels. For example in generic Technicolour models in which vector and axial-vector composite states can have the aforementioned property or also in supersymmetric models with two nearby neutralinos. Other examples are models in warped extra dimension, like gauge-phobic Higgs models and Composite Higgs models.

As a conclusion, the interference effects can be crucial to study the phenomenology of such models at the LHC, and to determine its discovery potential. We stress that a proper treatment should be carefully and systematically implemented into Monte Carlo generators used to study BSM models, as physical results may be dramatically different from a naive use of the Breit-Wigner approximation.

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A Appendix: exact amplitudes

We give in this appendix some detailed formulae which were not given or given in approximate form in the text. Defining:

$$\lambda(p, m_A, m_B) = \frac{(p^2 - m_B^2 + m_A^2)^2 - 4p^2 m_A^2}{p^4}\,, \tag{A.1}$$

the imaginary contribution to of the 1-PI corrections of the scalar amplitudes are, at one-loop level, (m_A and m_B are the masses of the particles in the loop):

$$\begin{aligned}
\Sigma_f^{(s)}(p^2) &= \frac{\sqrt{\lambda(p, m_A, m_B)}}{16\pi}\left[(\lambda_L^i\lambda_R^j + \lambda_R^i\lambda_L^j)(p^2 - m_A^2 - m_B^2) + \right. \\
&\quad \left. -2(\lambda_L^i\lambda_L^j + \lambda_R^i\lambda_R^j)m_A m_B\right]\,, &\text{(A.2)}\\
\Sigma_s^{(s)}(p^2) &= \frac{\sqrt{\lambda(p, m_A, m_B)}}{16\pi}\lambda_s^i\lambda_s^j\,, &\text{(A.3)}\\
\Sigma_V^{(s)}(p^2) &= \frac{\sqrt{\lambda(p, m_A, m_B)}}{64\pi}\lambda_V^i\lambda_V^j\left[\frac{(p^2 - m_A^2 - m_B^2)^2}{m_A^2 m_B^2} + 8\right]\,. &\text{(A.4)}
\end{aligned}$$

In a similar way for the vector amplitudes we have:

$$\begin{aligned}
\Sigma_f^{(V)}(p^2) &= \frac{\sqrt{\lambda(p, m_1, m_2)}}{16\pi}\left[\frac{\lambda_L^i\lambda_L^j + \lambda_R^i\lambda_R^j}{3}\left(2p^2 - m_A^2 - m_B^2 - \frac{(m_A^2 - m_B^2)^2}{p^2}\right) + \right. \\
&\quad \left. +(\lambda_L^i\lambda_R^j + \lambda_R^i\lambda_L^j)2m_A m_B\right]\,, &\text{(A.5)}\\
\Sigma_s^{(V)}(p^2) &= \frac{\sqrt{\lambda(p, m_A, m_B)}}{16\pi}\frac{\lambda_s^i\lambda_s^j}{3}\left[p^2 - 2(m_A^2 + m_B^2) + \frac{(m_A^2 - m_B^2)^2}{p^2}\right]\,, &\text{(A.6)}\\
\Sigma_V^{(V)}(p^2) &= \frac{(\lambda(p, m_A, m_B))^{3/2}}{192\pi}\lambda_V^i\lambda_V^j\frac{p^6}{m_A^2 m_B^2} \\
&\quad \cdot\left[1 + 10\frac{m_A^2 + m_B^2}{p^2} + \frac{m_A^4 + m_B^4 + 10 m_A^2 m_B^2}{p^4}\right]\,. &\text{(A.7)}
\end{aligned}$$

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